Rotating Earth

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(Effect of Centrifugal and Coriolis force due to Earth Rotation):

As we know that the earth is rotating uniformly about its own axis from west to East and a reference frame fixed on its surface is clearly a rotating frame of reference. Consequently, the two fictious forces such as Centrifugal force and Coriolis force must act on a particle at rest and in motion respectively relative to the rotating frame of reference.

Effect of Centrifugal force on g.

The acceleration of a particle in a uniformly rotating frame is specified as

$$\vec{a} = \vec{a}' + 2\vec{\omega} \times \vec{v}' + \vec{\omega} \times (\vec{\omega} \times \vec{r}) \quad \text{—} (1)$$

If a particle is at rest in the rotating frame then the Coriolis acceleration is given by $2\vec{\omega} \times \vec{V}' = 0$.

If body is under the action of only Centrifugal force then it is given by $\vec{\omega} \times (\vec{\omega} \times \vec{r})$.

Hence, the acceleration of the particle with respect to the inertial frame will be given by —

 $\vec{a} = \vec{a}' + \vec{\omega} \times (\vec{\omega} \times \vec{r})$ — (2)

or $\vec{a}' = \vec{a} - \vec{\omega} \times (\vec{\omega} \times \vec{r})$

Since earth is rotating from west to east and the reference frame attached to it is the rotating frame of reference. Let us take the angular velocity vector $\vec{\omega}$ along the Y-axis and the x-axis perpendicular to it as shown in figure to consider the effect of the centrifugal force on the acceleration due to gravity, then at a point P in latitude λ the actual value of $\vec{g}$ will act along the direction PO where O is the centre of the earth. The components of $\vec{g}$ are given as;

$-g\cos\lambda\,\hat{i}$ along x-axis and $-g\sin\lambda\,\hat{j}$ along Y-axis.

$\therefore\ \vec{g} = -g(\cos\lambda\,\hat{i} + \sin\lambda\,\hat{j})$

Also, $\vec{\omega} = \omega\hat{j}$

~~From~~

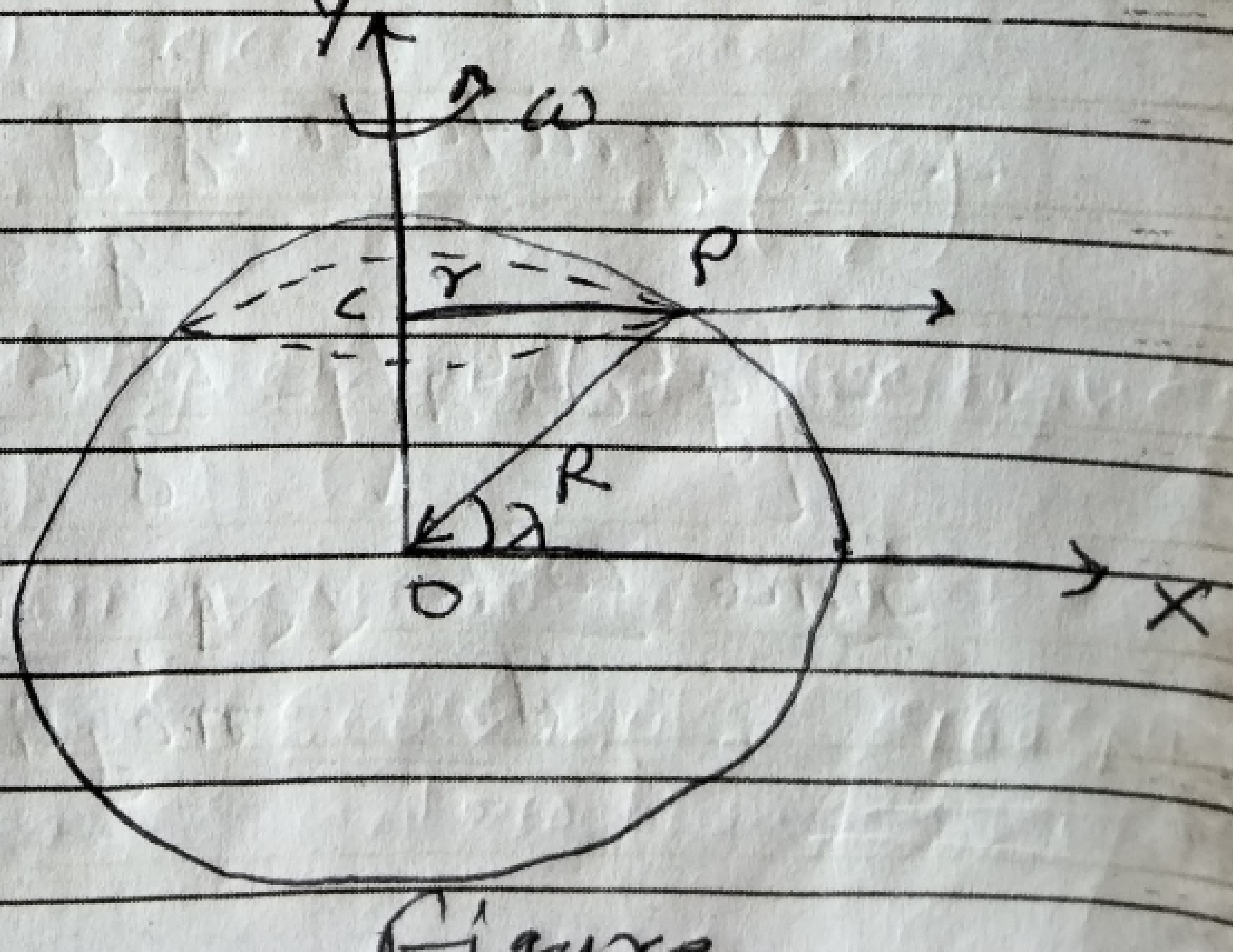

figure.

From point P draw a perpendicular PC at OY Then $CP = \vec{r}$. Here $\hat{i}$ denotes the radius of the circle in which a particle at P rotates.

Hence, $CP = \vec{r} \equiv \vec{R}\cos\lambda = R\cos\lambda\,\hat{i}$
where R = magnitude of the radius of the earth.

Taking into consideration, the effect of centrifugal acceleration and neglecting the effect of Coriolis acceleration due to Earth's rotation, the observed value of acceleration due to gravity at P is represented as ;

$$\vec{g}' = \vec{g} - \vec{\omega} \times (\vec{\omega} \times \vec{r})$$
$$= -g(\cos\lambda\,\hat{i} + \sin\lambda\,\hat{j}) - \omega\hat{j} \times (\omega\hat{j} \times R\cos\lambda\,\hat{i})$$
$$= -\left[(g\cos\lambda - \omega^2 R\cos\lambda)\,\hat{i} + g\sin\lambda\,\hat{i}\right]$$

$\therefore$ magnitude of $g' = \left[(g\cos\lambda - \omega^2 R\cos\lambda)^2 + g^2\sin^2\lambda\right]^{1/2}$

$$= \left[g^2\cos^2\lambda\left(1 - \frac{\omega^2 R}{g}\right)^2 + g^2\sin^2\lambda\right]^{1/2}$$

Since $\frac{\omega^2 R}{g} << 1$, its square is a negligible quantity.

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$$\therefore g' = g\left[\cancel{\cos^2\lambda - \frac{2\omega^2 R\cos^2\lambda}{g} + \sin^2\lambda}\right]^{1/2}$$

$$= g\left[1 - \frac{2\omega^2 R}{g}\cos^2\lambda\right]^{1/2} = g\left[1 - \frac{\omega^2 R}{g}\cos^2\lambda\right]$$

$$= g - \omega^2 R\cos^2\lambda$$

Taking $R = 6.38 \times 10^6 m$, The value of

$$R\omega^2 = 6.38 \times 10^6 \times \left(\frac{2\pi}{24 \times 60 \times 60}\right)$$

$$= 0.03372\, ms^{-1} = 3.372\, cm/sec^2$$

At The Equator :

At The equator $\lambda = 0$, $\cos\lambda = 1$

$\therefore g' = g - \omega^2 R\cos^2\lambda = g - \omega^2 R = g - 3.372\, cm/sec^2$

Hence The value of g at The equator decreases by $3.372\, cm/sec^2$ because of Earth's rotation.

At The Poles :

At Poles $\lambda = 90°$, $\cos\lambda = 0$. Hence The value of g decreases because rotation is Zero. The observed difference between The value of g at The Poles and at The equator is about $5.2\, cm/sec^2$. This is because The fact that The Earth is flat at The Poles and even if it did not rotate, The value of g at The Poles would be greater as compared to equator.